Technical Notes

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Optimal Frequency Separation of Cylindrical Shells

Michael Pappas*
New Jersey Institute of Technology, Newark, N.J.

I. Introduction

BRONOWICKI et al[1,2] in their extension of earlier studies of optimal synthesis of "T" ring stiffened cylindrical shells under hydrostatic pressure[3-5] introduce the problem of optimal natural frequency separation in such shells. Unfortunately, the form of gradient-based mathematical programming optimization procedure they employed does not appear capable of solving the formulated problem.

Mathematical programming procedures search out those values $\bar{x}_i$ of the variables x_i that result in

$$f(\bar{x}_i) = \min f(x_i) \qquad i=1,2,\ldots I \tag{1}$$

subject to the constraints

$$g_j(x_i) \geq 0 \qquad j=1,2,\ldots J \tag{2}$$

Consider the problem of increasing the separation between the lowest, ω_1, and the second lowest, ω_2, natural frequency in the neighborhood of a point x_i^r where $\omega_2 \simeq \omega_3$, the third lowest frequency. Gradient-based procedures such as those used in Ref. 1 determine a direction for function improvement at some point x_i^r from the derivatives of ω_1 and ω_2 with respect to x_i^r. A move, based on some gradient strategy, in a direction increasing the separation between the frequencies associated with these modes may, however, reduce the separation between the frequencies associated with the modes that produced ω_1^r and ω_3^r. Where $\omega_2 \simeq \omega_3$, these latter modes could then produce the two lowest frequencies after a move to point x_i^{r+1} where the separation then would be lower than at point x_i^r, thus producing algorithm failure. A procedure is needed at such points that will separate ω_1 and ω_2 while simultaneously separating ω_1 and ω_3.

II. Method of Solution

The DSFD procedure[6,7] is adaptable to the frequency separation problem. This procedure couples a direct search scheme with a gradient-based direction-finding procedure used at points of direct search failure.

As the direct search avoids the use of derivatives, the objective function given by

$$f(x_i) = \omega_1 - \omega_2 \tag{3}$$

can be used without difficulty at all points where direct search is utilized.

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*Associate Professor of Mechanical Engineering, Newark College of Engineering. Member AIAA.

At points of direct search failure a modified version of the direction-finding procedure of Zoutendijk[8] is utilized to restart the basic direct search. A direction s_i in which an improved point may be found is normally determined from the solution of the linear programming problem:

Given the set x_i, find the set s_i that results in a

$$\max \sigma \tag{4}$$

for which

$$\sigma > 0 \tag{5}$$

$$(s_i)^T \nabla f(x_i) + \sigma \leq 0 \tag{6}$$

$$-(s_i)^T \nabla g_i(x_i) + W_j \sigma \leq 0 \quad j \epsilon J_a \tag{7}$$

$$-1 \leq s_i \leq 1 \tag{8}$$

Here $(s_i)^T$ indicates the transpose of vector s_i, W_j is a weighting parameter, and the set J_a contains the active constraints.

At points where

$$\omega_3 - \omega_2 < e \tag{9}$$

and e is an arbitrary small constant indicating frequency similarity, Eq. (6) is replaced by the set

$$(s_i)^T \nabla f_1(x_i) + \sigma < 0$$

$$(s_i)^T \nabla f_2(x_i) + \sigma < 0 \tag{10}$$

where f_1 is given by Eq. (3) and

$$f_2 = \omega_1 - \omega_3 \tag{11}$$

It may be seen that the solution to the modified direction-finding problem will yield a direction that will simultaneously separate ω_1 and ω_3 as well as ω_1 and ω_2.

In the event $\omega_2 \approx \omega_3 \approx \omega_4$ this modification procedure may be extended by replacing Eq. (6) by three equations of the form of Eqs. (10), thereby generating a direction that separates ω_1 and ω_4 as well as ω_1 and ω_2 and ω_1 and ω_3.

III. Shell Design Problem

An optimal shell design capability was developed along the lines of earlier procedures[3,4] to allow a study of the aforementioned optimal frequency separation method. Two problem types were studied.

Type A Problem

The objective function here is given by Eq. (3). The constraints used are: g_1 = gross (general) buckling, g_2 = shell (inter-ring) buckling, g_3 = shell yielding, g_4 = stiffener yielding, g_5 = stiffener flange buckling, g_6 = maximum flange thickness, g_7 = minimum flange width, g_8 = minimum internal or maximum external radius, g_9 = minimum natural frequency, g_{10} = maximum weight, and g_{11} = web buckling. This is the optimal frequency separation problem (type II) of Refs. 1 and 2.

The equations used for g_3, g_4, g_6, g_7, and g_8 are taken from Ref. 2. Those used for g_5 and g_{11} are essentially Eqs. (11) of Ref. 2. Reference 2 uses the same equation as Ref. 3 for the shell and stiffener yield constraints g_3 and g_4 except that Ref. 2 ignores the effect of eccentricity (out-of-roundness) on stiffener stress. The basic behavior prediction equations for constraints g_2 and g_9 are adapted from Ref. 5 which uses a procedure described in Ref. 9.

It should be noted that the use of an in vacuo model used here and in Refs. 1 and 2 for the vibration of shells submerged in water may produce substantial error.[10] However, its use here is justified in light of the objective of this work, which is to present and evaluate the effectiveness of new methodology for the design of shells with optimal frequency separation.

A comparison of the values of the inter-ring buckling pressures produced by Eq. (8) of Ref. 3 and the procedure used here shows values within 2% for the designs of this study. Since the validity of the equation used in Ref. 3 has been demonstrated by experiment[11] the model used here for inter-ring buckling appears to be suitable even though it assumes simple support of the ends of the inter-ring panel. The effect of this assumption on the natural frequency values for such short shells is unknown. The use of this model for natural frequency prediction in this preliminary study, however, seems justified in light of the rather substantial error produced by the in vacuo assumption and the objectives of the study. A consideration of inter-ring vibration allows an opportunity to determine if this mode appears to be significant for the types of problems studied here. Furthermore, inclusion of this vibration mode avoids the inconsistency in Ref. 1 wherein both an inter-ring buckling constraint (natural frequency is zero where such a constraint is active) and a minimum natural frequency constraint are specified.

Type B Problem

The objective in this problem type is to maximize the lowest natural frequency. Thus the objective function here is

$$f(x_i) = -\omega_1 \tag{12}$$

This problem type uses the same constraints as type A.

A difficulty similar to that arising in problem type A is encountered here. The optimal search will encounter points where $\omega_1 \simeq \omega_2$. This situation is treated in the same fashion as for the type A problem by letting $f_1 = -\omega_1$, $f_2 = -\omega_2$, and using Eqs. (10) in the direction-finding problem.

IV. Results

A computer program called SBSHL7 coupling the DSFD optimization procedure of Ref. 6 and problems just described was used to repeat some of the shell design studies of Refs. 1, 3, and 7 in order to evaluate its performance on the optimal frequency separation problem. All studies presented here use the design parameters of Ref. 1.

Problem type A and B were run using five and six variable formulations.[7] Table 1 gives the results of typical problem type A and B runs. A maximum weight displacement ratio (W_D) of 0.150 was used for these runs. The quantities n_f and n_g are the number of objective and constraint function evaluations required for convergence.

An examination of the optimal design for the type A problem reveals several interesting characteristics. Not only is ω_3 essentially equal to ω_2, as found in Refs. 1 and 2, but ω_4 is also essentially equal to ω_2. Furthermore, ω_4 is associated with shell panel (inter-ring) vibration. The design is characterized by relatively large-frame spacing and large, deep framing members. Thus, the optimal design uses the largest frame members possible without inducing a shell panel vibration mode lower than the second frequency associated with the gross vibration, and without violating the maximum weight constraint. It is apparent, therefore, that the shell panel vibration should be considered in optimal frequency separation problems. Furthermore, optimal

Table 1 Designs for problem types A and B

	Initial design	Type A, 5 var. optimum	Type A, 6 var. optimum	Type II, Ref. 1	Type B, 6 var. optimum
W_D	0.155	0.150	0.150	0.135	0.150
Skin thickness,	25.40	38.46	38.56	31.03	35.00
mm (in.)	(1.000)	(1.514)	(1.518)	(1.222)	(1.378)
Web thickness,	25.40	13.97	13.69	10.03	11.81
mm (in.)	(1.000)	(0.550)	(0.539)	(0.395)	(0.465)
Flange thickness,	25.40	15.29	33.81	11.82	23.04
mm (in.)	(1.000)	(0.602)	(1.331)	(0.465)	(0.907)
Flange width,	254.0	246.4	122.6	445.8	541.0
mm (in.)	(10.000)	(9.702)	(4.826)	(17.55)	(21.30)
Stiffner spacing,	508.0	1396	1398	859.9	1372
mm (in.)	(20.000)	(54.96)	(55.04)	(33.85)	(54.00)
Web height,	254.0	820.9	804.9	526.3	731.0
mm, (in.)	(10.000)	(32.32)	(31.69)	(20.72)	(28.78)
g_1	0.539	0.789	0.790	0.677	0.749
g_2	0.472	0.304	0.307	0.342	0.126
g_3	0.001	0.114	0.116	−0.002	0.001
g_4	0.389	0.467	0.466	0.325	0.521
g_5	0.947	0.868	0.994	0.619	0.714
g_6	0.000	0.602	0.113	0.018	0.341
g_7	10.000	9.702	4.826	17.551	21.998
g_8	0.598	0.544	0.543	0.574	0.553
g_9	0.534	0.717	0.717	0.673	0.743
g_{10}	−0.035	0.001	0.001	0.120	0.000
g_{11}	0.967	0.000	0.000	−0.006	0.005
$\omega_1 (n_1 m_1)$ Hz	25.76(2,1)	42.40(2,1)	42.41(2,1)	28.37(36.65)[b]	46.71(2,1)
$\omega_2 (n_2 m_2)$ Hz	33.04(3,1)	72.84(1,1)	72.90(3,1)	51.96(58.52)[b]	46.87(14,1)[a]
$\omega_3 (n_3 m_3)$ Hz	44.29(3,2)	72.84(3,1)	72.90(1,1)	51.96	47.50(15,1)[a]
$\omega_4 (n_4 m_4)$ Hz	47.82(1,1)	72.88(14,1)[a]	73.19(14,1)[a]	—	49.83(13,1)[a]
$(\omega_2 - \omega_1)$ Hz	7.28	30.44	30.49	23.59(21.86)[b]	0.16
n_f	1	2963	16,779	—	22,572
n_g	11	2118	34,371	—	50,193

[a] Mode associated with shell (inter-ring) vibration. [b] The first value is that reported in Ref. 1. The quantity in parentheses is the frequency computed using the Donnell shell theory employed here.

separation is achieved by paying the maximum penalty in weight, a conclusion which differs from Refs. 1 and 2.

The characteristic of the design with the maximum lowest frequency is rather similar to that with the largest frequency separation. The former has a somewhat higher lowest frequency but no significant separation of this frequency from the second or third lowest frequencies. Here again, shell panel vibration controls the design.

Comparing the designs in Table 1, it appears that rather similar designs may behave quite differently with respect to frequency separation. This raises a serious question with regard to the validity of employing the frequency prediction models used here for design for optimal frequency separation of immersed shells. Orthotropic shell theory does not provide an extremely accurate approximation to actual behavior. Furthermore, this and the earlier studies of Refs. 1 and 2 use in vacuo frequencies to study the characteristics of submerged shells. Considering the inaccuracy in these models and the sensitivity of the frequency separation results to relatively small design changes, the results do not appear to be useful for design purposes.

In addition to the search starting point cited in Table 1, runs were made using the six variable formulations for problem types A and B from the three additional starting points. These also converged to designs similar to those in Table 1; thus the starting point sensitivity noted in Ref. 1 on their problem type B was not apparent in this study. These results indicate that the optimization procedure used here is apparently capable of locating an optimum design with reasonable reliability for the problem types studied.

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A Minimum Mass Square Plate with Fixed Fundamental Frequency of Free Vibration

M. H. Foley*
Clarkson College of Technology, Potsdam, N. Y.

Introduction

THE minimum mass design of structural members subject to dynamic behavioral constraints has been the focus of considerable research effort recently.[1] Constraints on one or more natural frequencies of free vibration or flutter speed are often imposed in the design of major structural components for lightweight, high-performance aircraft and aerospace vehicles.

One approach to the minimum mass design of basic structural members, such as a beam, a column, or a plate, is to represent the member by a continuous model whose behavior is described by a differential equation. The optimal design is determined by applying methods based on the calculus of variations or its extension in the form of optimal control theory.

The application of methods from the theory of optimal control has proven to be a very powerful technique when the nature of the structural member is such that its behavior can be described by an ordinary differential equation in one independent spatial variable (one-dimensional structure). Relatively few applications have been made to the optimal design of structural members whose behavior is described by a partial differential equation in two independent spatial variables (two-dimensional structures). In control theory terminology, these would be classified as distributed parameter optimal control problems.

The purpose of this Note is to illustrate the application of a simple computational technique to the problem of determining the minimum mass design of a simply supported square plate with fixed fundamental frequency of free vibration. Previous efforts to solve this problem[2-4] have utilized methods requiring the numerical solution of a boundary value problem characterized by the partial differential equation modeling the behavior of the plate. This is an extremely complex and time-consuming process. One theoretically promising technique for solving optimal control problems that avoids explicitly solving the governing differential equation for the system is the ϵ method of Balakrishnan.[5] The basic idea of this method is to replace the differential equation modeling the system by a penalty function, thus transforming the original dynamic problem into a nondynamic one. Foley and Citron[6] have applied a technique based on the ϵ method to one-dimensional structural optimization problems. This Note presents an extension of that technique to a two-dimensional structural optimization problem.

Formulation of the Optimal Design Problem

The design objective is to continuously vary the thickness, $T(X,Y)$, of the plate over the region $\{(X,Y) \mid 0 \le X \le a,\quad 0 \le Y \le a\}$ so as to minimize the mass of the plate, while keeping the fundamental frequency of free vibration fixed and equal to that of a reference plate of uniform thickness, T_0, and identical dimensions. The behavior of the plate may be

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*Assistant Professor, Department of Mechanical Engineering.